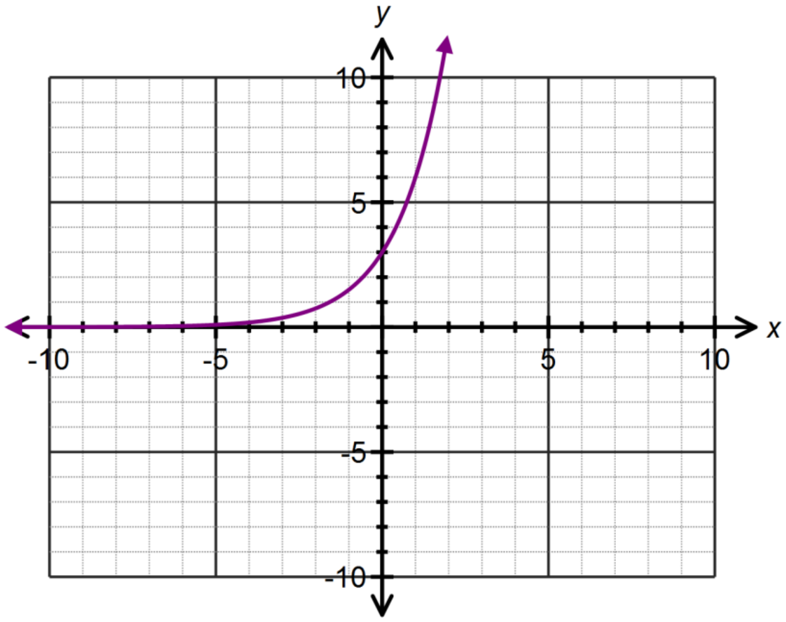
Calculator Free
Logarithmic Graphs and Differentiation

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 3, 2 = 7 marks] **CF**

Consider the exponential function drawn below.



(a) State the equation of the exponential function in the form $y = a \times b^x$.

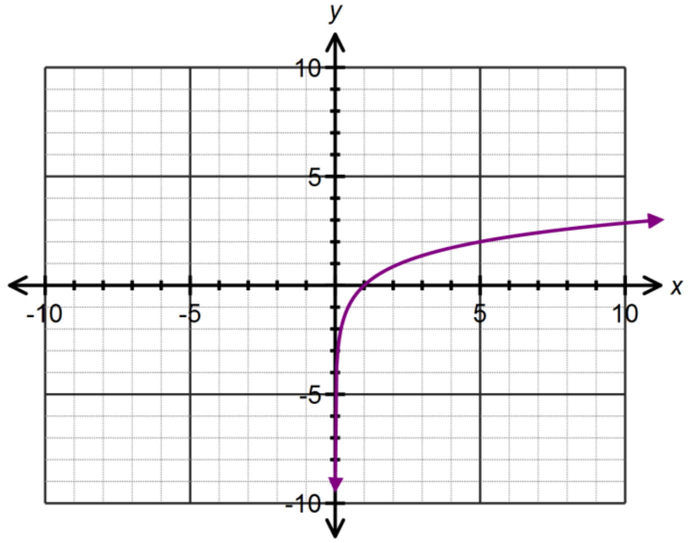
(b) Use the exponential graph drawn, and an appropriate mirror line, to draw the logarithmic function which is the inverse of the given exponential function.

(c) Hence or otherwise determine the equation of the logarithmic function, $y = \log_a(bx)$ which is the inverse of the given exponential function with the same base.

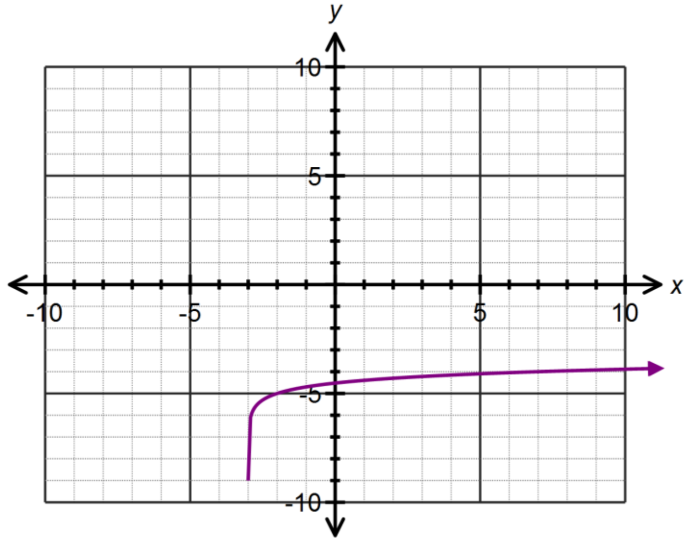
Question Two: [2, 3, 3 = 8 marks] CF

Determine the equation of each of the following graphs drawn below:

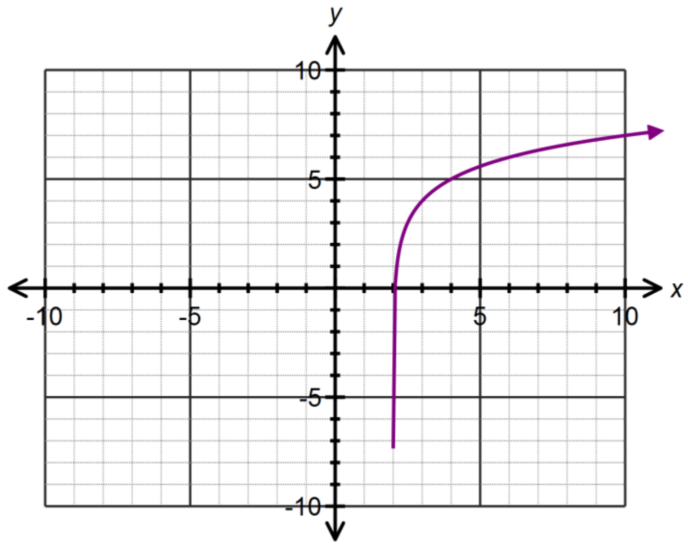
(a)



(b)

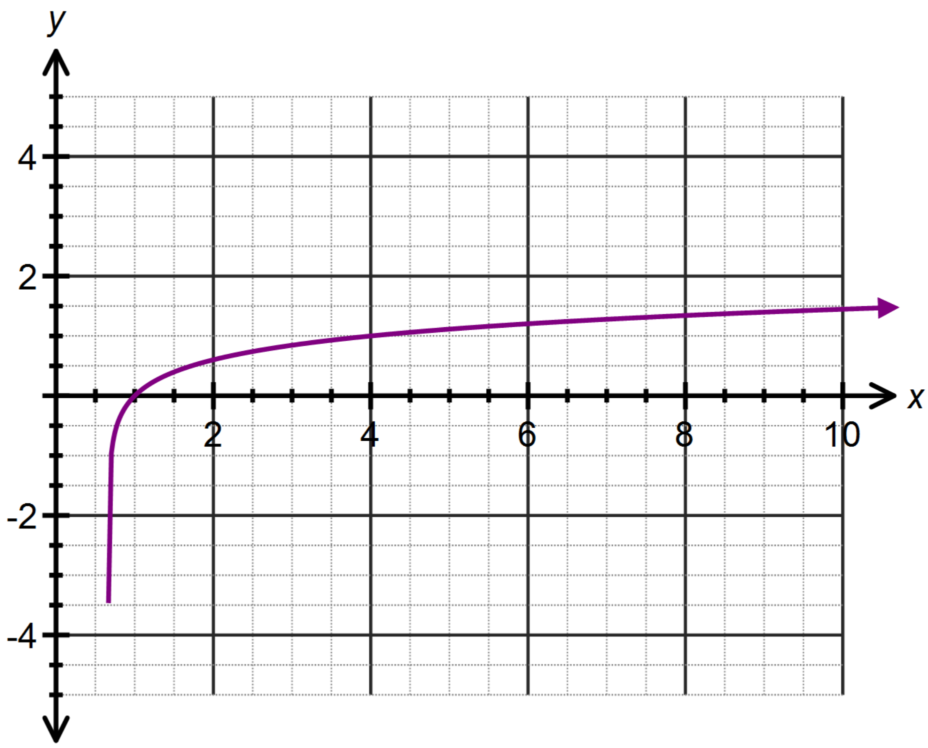


(c)



Question Three: [2, 1, 2 = 5 marks] CF

The function $f(x) = \log(ax - 2)$ is drawn below.



(a) Determine the value of *a*.

(b) Use the graph to approximate the solution to $\log(ax - 2) = -1$

(c) Solve $\log(ax - 2) = 2$ algebraically.

Question Four: **[1, 3, 3, 2, 3, 3 = 15 marks]** **CF**

Differentiate each of the following with respect to *x*, showing full working:

(a) $y = \ln(4x - 5)$

(b) $f(x) = e^{1-x}\ln(\mathrm{x})$

(c) $g(x) = \ln\left(\dfrac{x^2}{\sqrt{x-1}}\right)$

(d) $y = \ln(\sin(3x))$

(e) $y = \log_2(x^3 - 2x)$

(f) $y = 5^x$

Question Five: [5, 5 = 10 marks] CF

(a) Determine the coordinates of the point(s) where the curve $y = \ln(2x-5)+1$ has a gradient of 2.

(b) Determine the equation of the tangent to the curve $y = x^2 \ln(x)$ at the point where $x = e$. Leave your answers as exact simplified values.

SOLUTIONS
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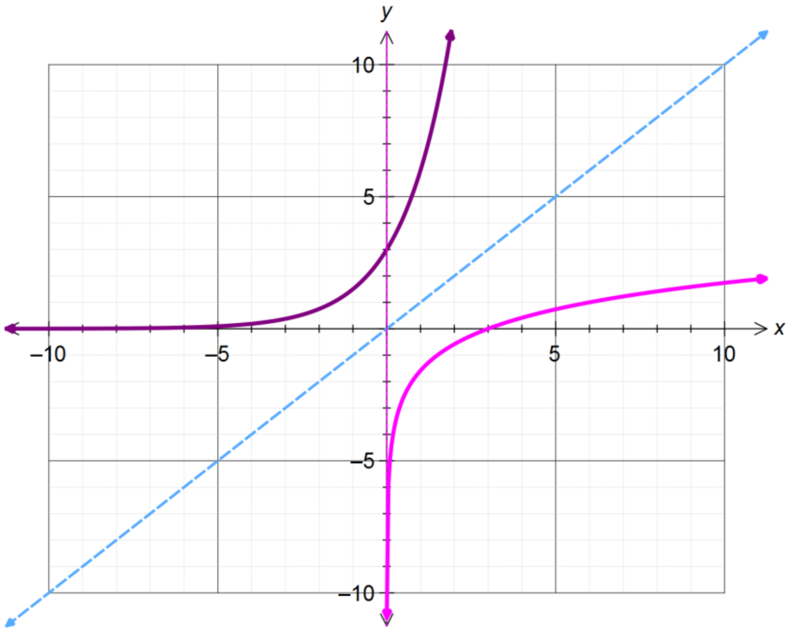
Question One: [2, 3, 2 = 7 marks] **CF**

Consider the exponential function drawn below.



(a) State the equation of the exponential function in the form $y = a \times b^x$.

$y = 3 \times 2^x$

(b) Use the exponential graph drawn, and an appropriate mirror line, to draw the logarithmic function which is the inverse of the given exponential function.

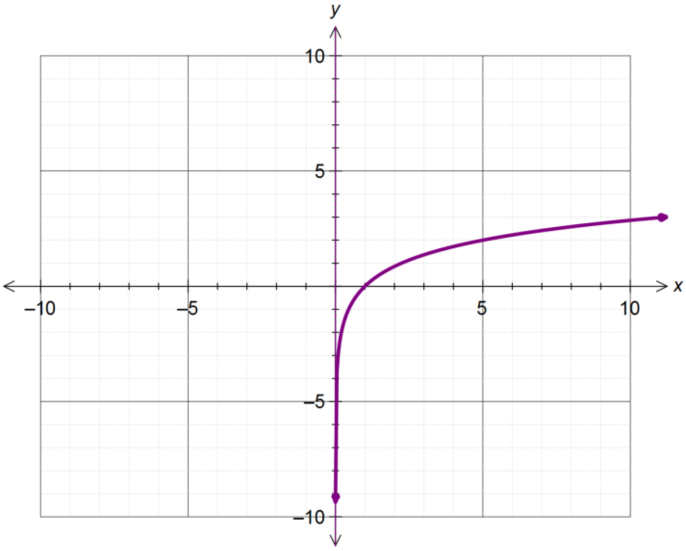
(c) Hence or otherwise determine the equation of the logarithmic function, $y = \log_a(bx)$ which is the inverse of the given exponential function with the same base.

$$y = \frac{\log(\frac{x}{3})}{\log 2} = \log_2\left(\frac{x}{3}\right)$$

Question Two: **[2, 3, 3 = 8 marks]** **CF**

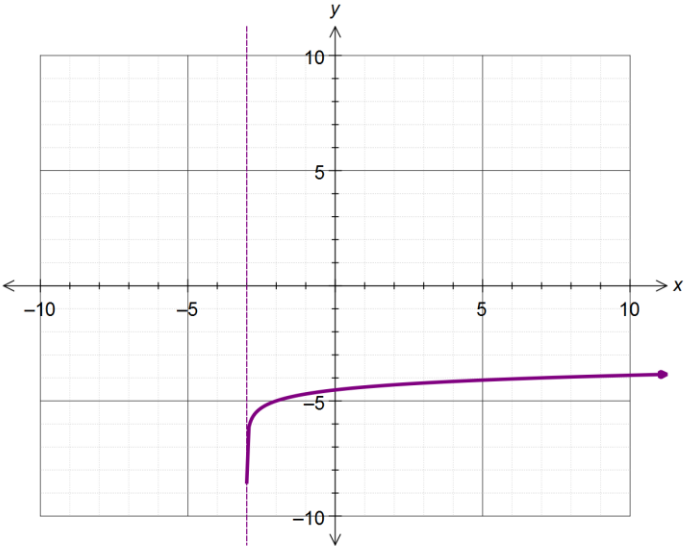
Determine the equation of each of the following graphs drawn below:

(a)



$y = 2\log_5 x$

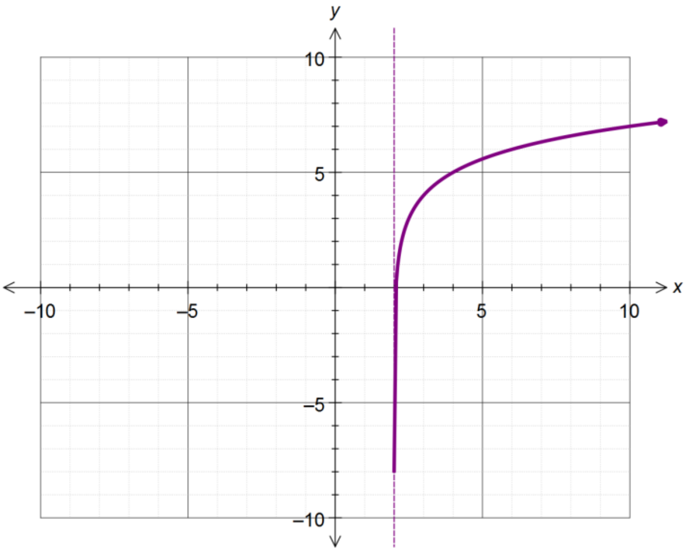
(b)



$y = \log(x+3) - 5$

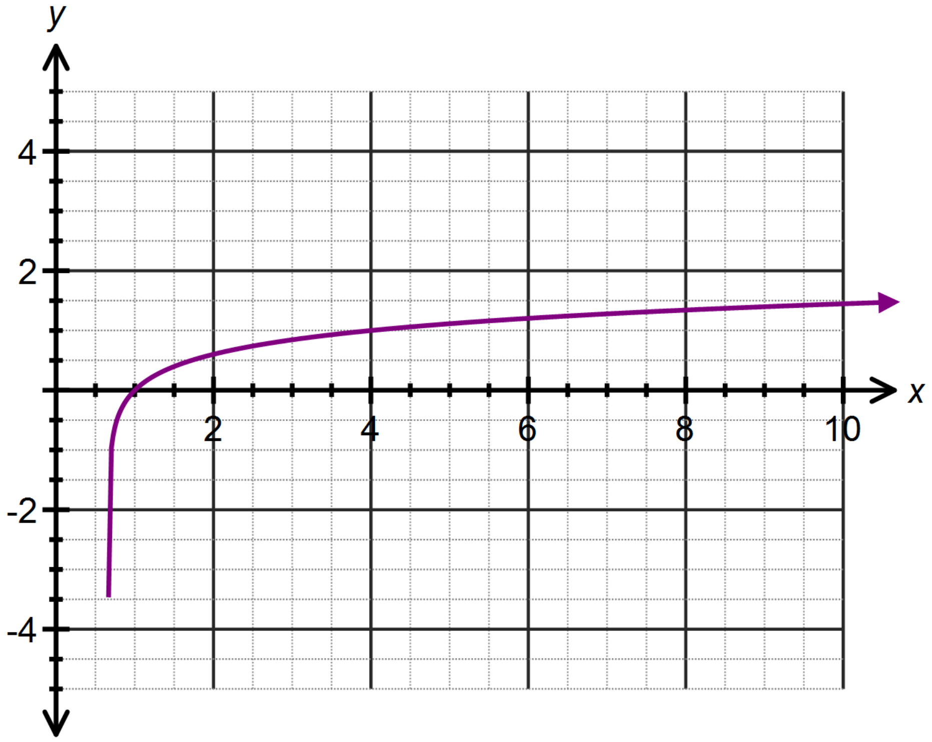
(c)

$y = \log_2(x-2) + 4$



Question Three: [2, 1, 2 = 5 marks] CF

The function $f(x) = \log(ax-2)$ is drawn below.



(a) Determine the value of a.

$0 = \log(a-2)$ ✓

$1 = a-2$

$3 = a$ ✓

(b) Use the graph to approximate the solution to $\log(ax-2) = -1$

$x \approx 0.8$ ✓

(c) Solve $\log(ax-2) = 2$ algebraically.

$\log(3x-2) = 2$ ✓

$3x-2 = 100$

$3x = 98$

$x = \dfrac{98}{3}$ ✓

Question Four: **[1, 3, 3, 2, 3, 3 = 15 marks]** **CF**

Differentiate each of the following with respect to x, showing full working:

(a) $y = \ln(4x-5)$

$$\frac{dy}{dx} = \frac{4}{4x-5}$$

(b) $f(x) = e^{1-x}\ln(x)$

$$f'(x) = -e^{1-x}\ln(x) + \frac{e^{1-x}}{x}$$

(c) $g(x) = \ln\left(\frac{x^2}{\sqrt{x-1}}\right)$

$$g(x) = \ln(x^2) - \frac{1}{2}\ln(x-1)$$

$$g'(x) = \frac{2}{x} - \frac{1}{2(x-1)}$$

(d) $y = \ln(\sin(3x))$

$$\frac{dy}{dx} = \frac{3\cos 3x}{\sin 3x}$$

(e) $y = \log_2(x^3-2x)$

$$y = \frac{\ln(x^3-2x)}{\ln 2}$$

$$\frac{dy}{dx} = \frac{3x^2-2}{(x^3-2x)\ln 2}$$

(f) $y = 5^x$

$$\ln y = x\ln 5$$

$$y = e^{x\ln 5}$$

$$\frac{dy}{dx} = \ln 5e^{x\ln 5} = \ln 5(5^x)$$

Question Five: [5, 5 = 10 marks] CF

(a) Determine the coordinates of the point(s) where the curve $y = \ln(2x-5)+1$ has a gradient of 2.

$$\frac{dy}{dx} = \frac{2}{2x-5}$$

$$\frac{2}{2x-5} = 2$$

$$2 = 4x - 10$$

$$x = 3$$

$$y = \ln(1) + 1 = 1$$

$$(3,1)$$

(b) Determine the equation of the tangent to the curve $y = x^2 \ln(x)$ at the point where $x = e$. Leave your answers as exact simplified values.

$$y = e^2 \ln(e) = e^2$$

$$\frac{dy}{dx} = 2x\ln(x) + \frac{x^2}{x}$$

$$\frac{dy}{dx} = 2x\ln(x) + x$$

$$\left.\frac{dy}{dx}\right|_{x=e} = 2e\ln(e) + e$$

$$\frac{dy}{dx} = 3e$$

$$y = 3ex + c$$

$$e^2 = 3e(e) + c$$

$$c = -2e^2$$

$$\therefore y = 3ex - 2e^2$$